Optimisation for the Telecommunication Industry using Quantum Annealing

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http://atadastral.co.uk/about/bt-labs/

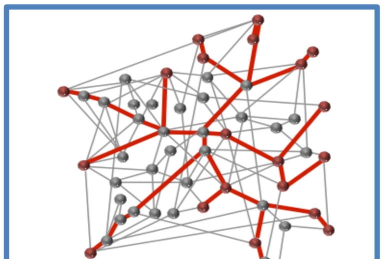
HARD PROBLEMS IN TELECOMMUNICATIONS

Resource allocation and planning problems in telecommunications are often algorithmically hard… (e.g. NP hard or #P complete)

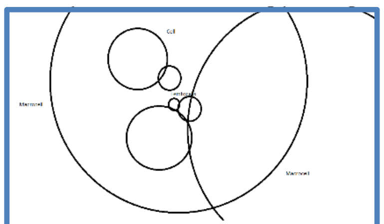
- Network layout problem (Steiner Tree)
- Job Scheduling
- Configuration of overlapping cells (placement, power, frequency assignment)
- Configurations of paths and wavelengths over core networks at layer 1 (RWA problem)

QCAPS Project

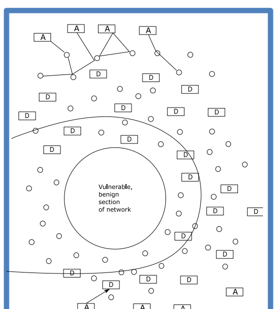
Hard Computational Problems from Telecommunications



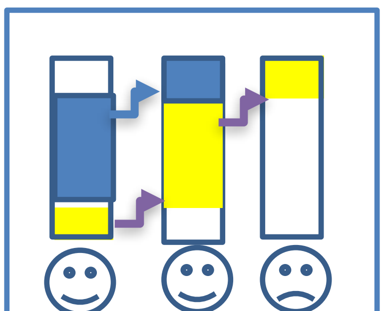
Steiner Tree



Graph Colouring



Max-Cut



Job Scheduling

D-Wave Approach to Optimisation (experiment)

- Using 2000Q D-Wave Processor
- Quantum Annealing
- Initial review of similar previous work by Nasa
- Selection of promising problems.

Overview of problems

Infrastructure layout	Using existing infrastructure such as ducts and poles, together with the possibility of creating new infrastructure, to provide or upgrade connectivity to new premises. (**Steiner spanning tree**) - *Simulated annealing approaches are typically used for these problems currently.* Location of cellular network base station.
Network capacity	Designing a network to have sufficient capacity to meet demand. Utilising an existing network to maximise capacity
Network resilience	Identification of paths with disjoint nodes and edges in a graph Design of a network such that there are multiple paths over disjoint nodes and edges between each pair of endpoints.
Network security	Optimum placement of deep packet inspection firewalls on a network Graph search problems – identification of similar or of unusual clusters (identification of suspicious behaviour)
Content distribution	Placement and size of content distribution nodes Predictive downloads to content distribution nodes
Network operation and maintenance	Location of service hubs Location and volumes of spare network components Priority and frequency of ‘uplift’ (scheduled, preventative maintenance) Allocation of jobs to engineers Geographic ordering of jobs (Travelling Salesman)

SUITABILITY FOR QUANTUM ANNEALING

- The problems described are discrete optimisation problems
- The state space is large, but can be represented in small number of bits
- Mapping the problem to qubits is tractable and allows us to find optimum and near optimum solutions.

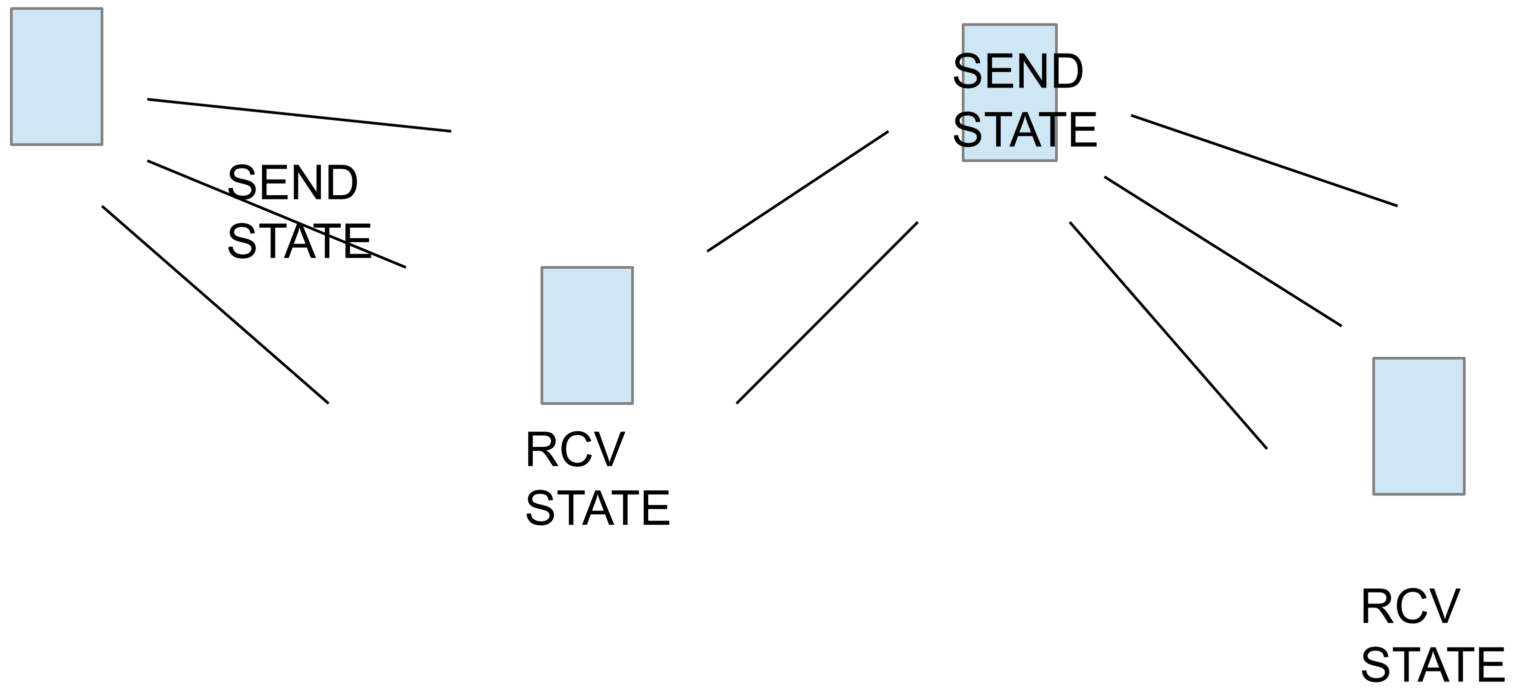
CELLULAR NETWORKS

- Coverage – placement and power of antenna
- Capacity – frequency assignment, interference management
- >These two issues may not be separable in the design problem
- Cell – Base Station power
- Cell shape, distance, multipath, obstacle shadowing, antenna characteristics
- SIR > threshold

MANET - Half duplex mesh network problem

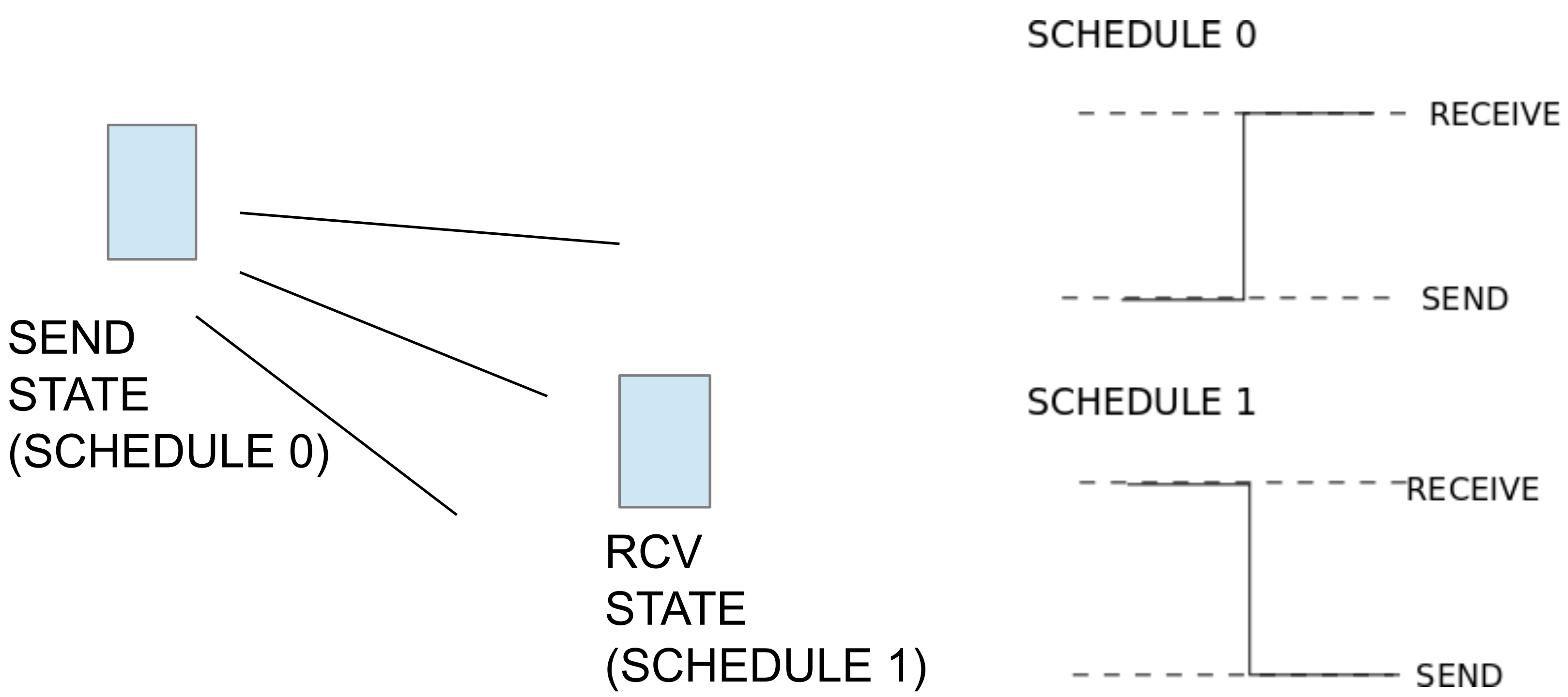
- MANET – Mobile Ad hoc Network.
- We consider a mesh network where intermediate devices relay data to provide complete communication services between all devices on the mesh.

- Applications: IoT, Environmental monitoring, Disaster areas



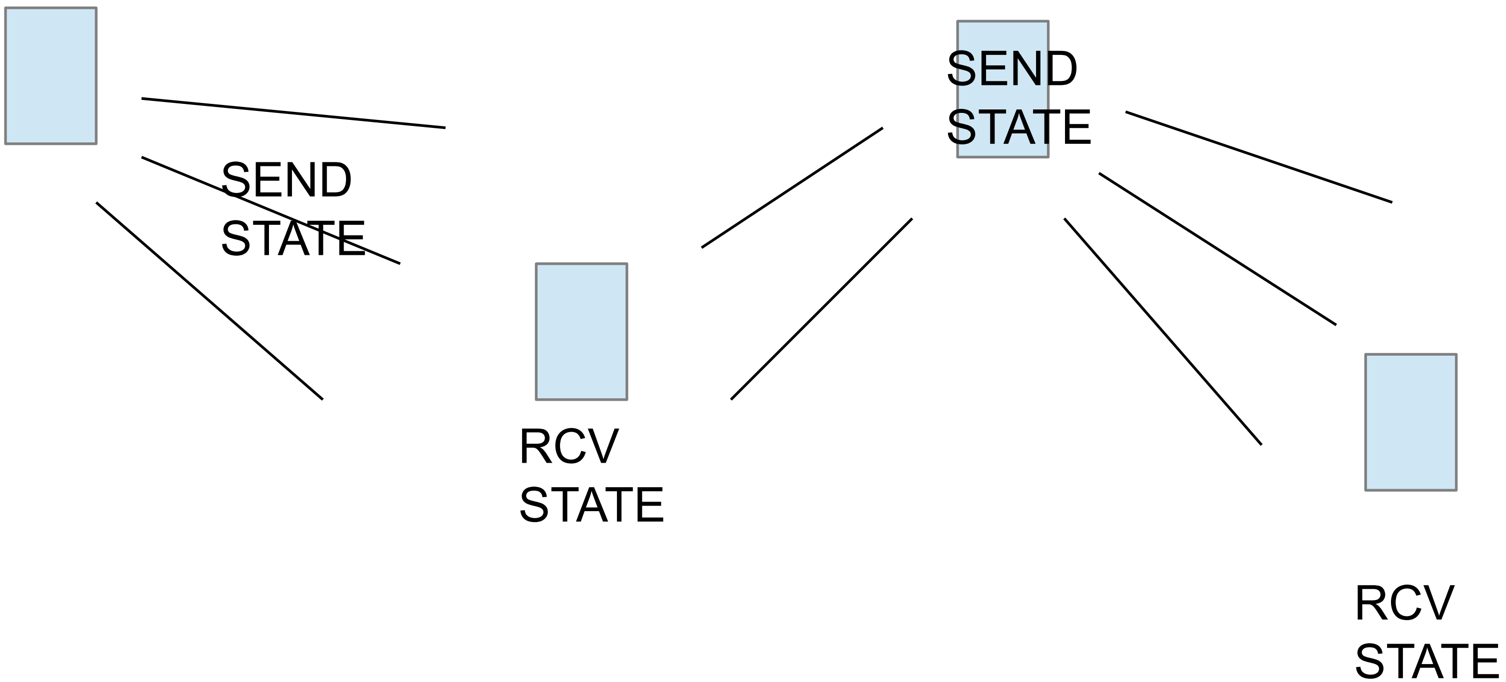
MANET - Half duplex mesh network problem

- Half-duplex problem (mesh network of devices which can either send or receive on a single frequency)
- Problem of finding an optimum schedule
- Assume devices ‘boot up’ in a sub-optimal schedule, and can communicate their discovered neighbours to a central ‘optimising service’ which will will communicate back a schedule.
- Devices can synchronise clocks.



MANET - Half duplex mesh network problem

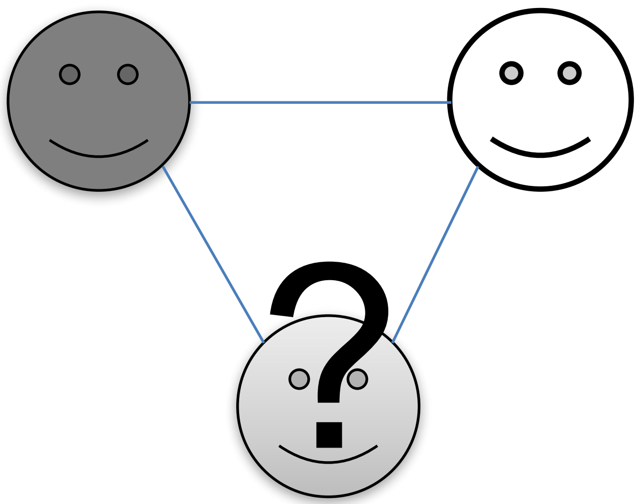
- Half-duplex problem maps very naturally to the D-Wave annealer Ising form.
- Simple 1 logical qubit per device is sufficient.
- SEND (SCHED 0) = -1
- RECEIVE (SCHED 1) = 1



MANET - Half duplex mesh network problem

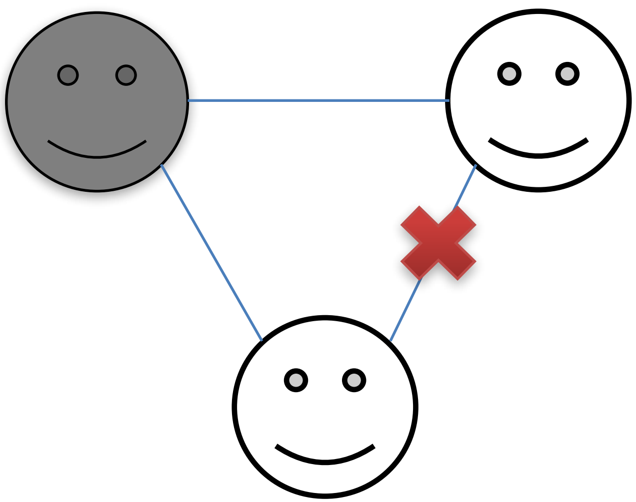
- 2 connected node problem

- 3 connected node problem – frustration.



MANET - Half duplex mesh network problem

- 3 connected node problem – a solution



- One link is disabled (frustrated link)
- (There will still need to be arbitration in the protocol, e.g. handshaking because one node communicates with two others)

MANET - Half duplex mesh network problem

- The code is straightforward – we pass the logical Ising Hamiltonian values in a JSON string

```
J = {(0, 1): 1, (1, 2): 1, (0, 2): 1}
h = [0, 0, 0]
```

- Dwave provides a classical function which handles the embedding of the logical Hamiltonian onto the physical qubits. (Although we can optionally run this many times, and select 'the best' embeddings')

```
# Get the geometry of the hardware
adj = get_hardware_adjacency(solver)
# Find an embedding for the problem.
emb = find_embedding(J, adj)
```

- Run the solver.

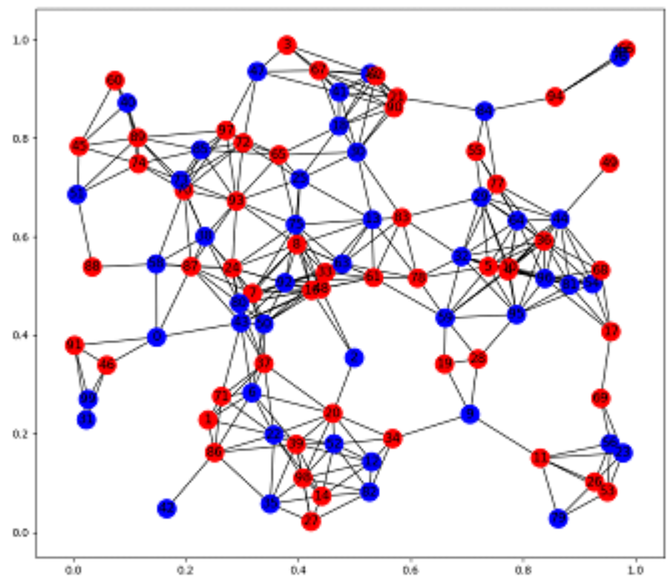
```
answer = solve_ising(solver, h_emb, J_emb, **dw_params)
```

- The results are returned as list of measured spin values for each qubit.

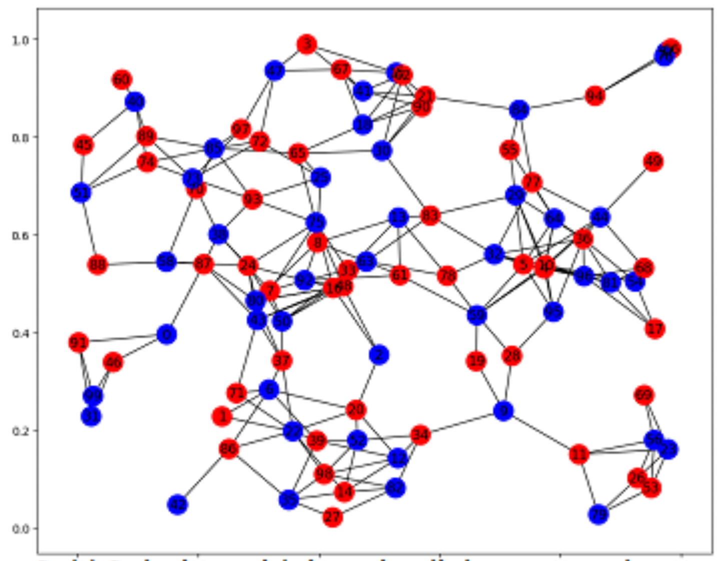
MANET - Half duplex mesh network problem

Optimising the Half Duplex Mesh is NP Hard!

- 2000Q architecture is suitable for embedding and solving 100 node mesh network problems of realistic graph density.



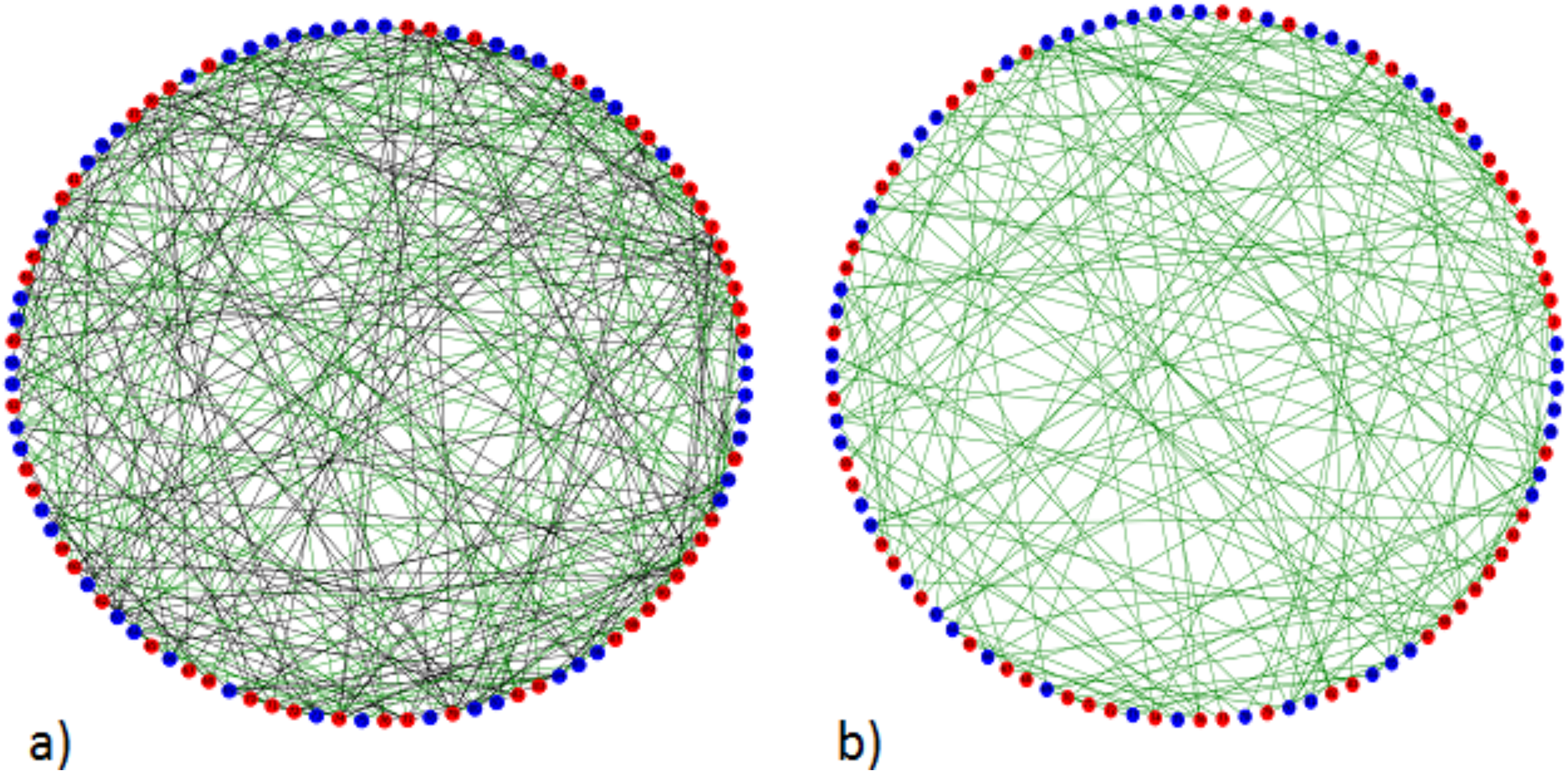
2.a) 100 node graph (2nd instance) - exact solution found by DWave



2. b) Solution with inactive links removed.

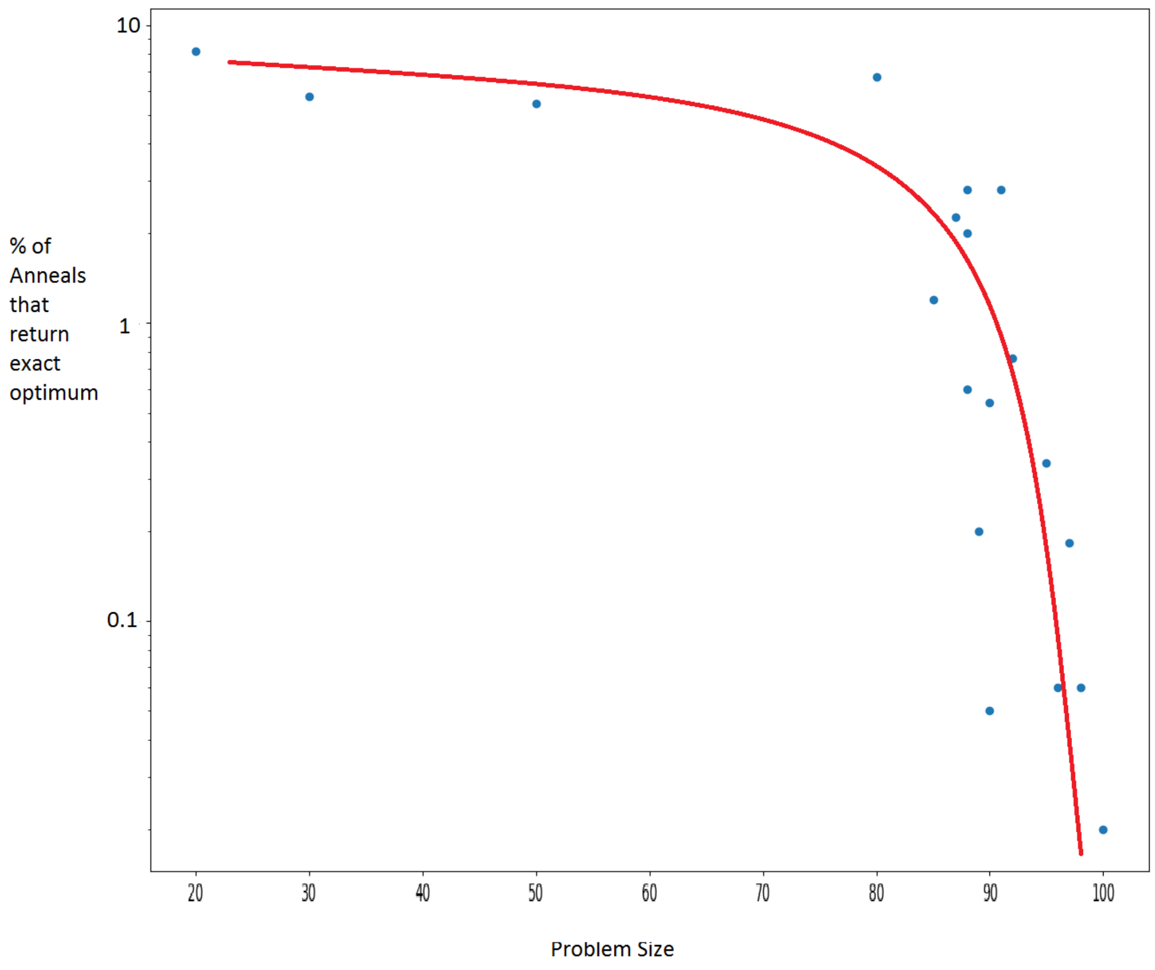
MANET - Half duplex mesh network problem

300 Node Graph



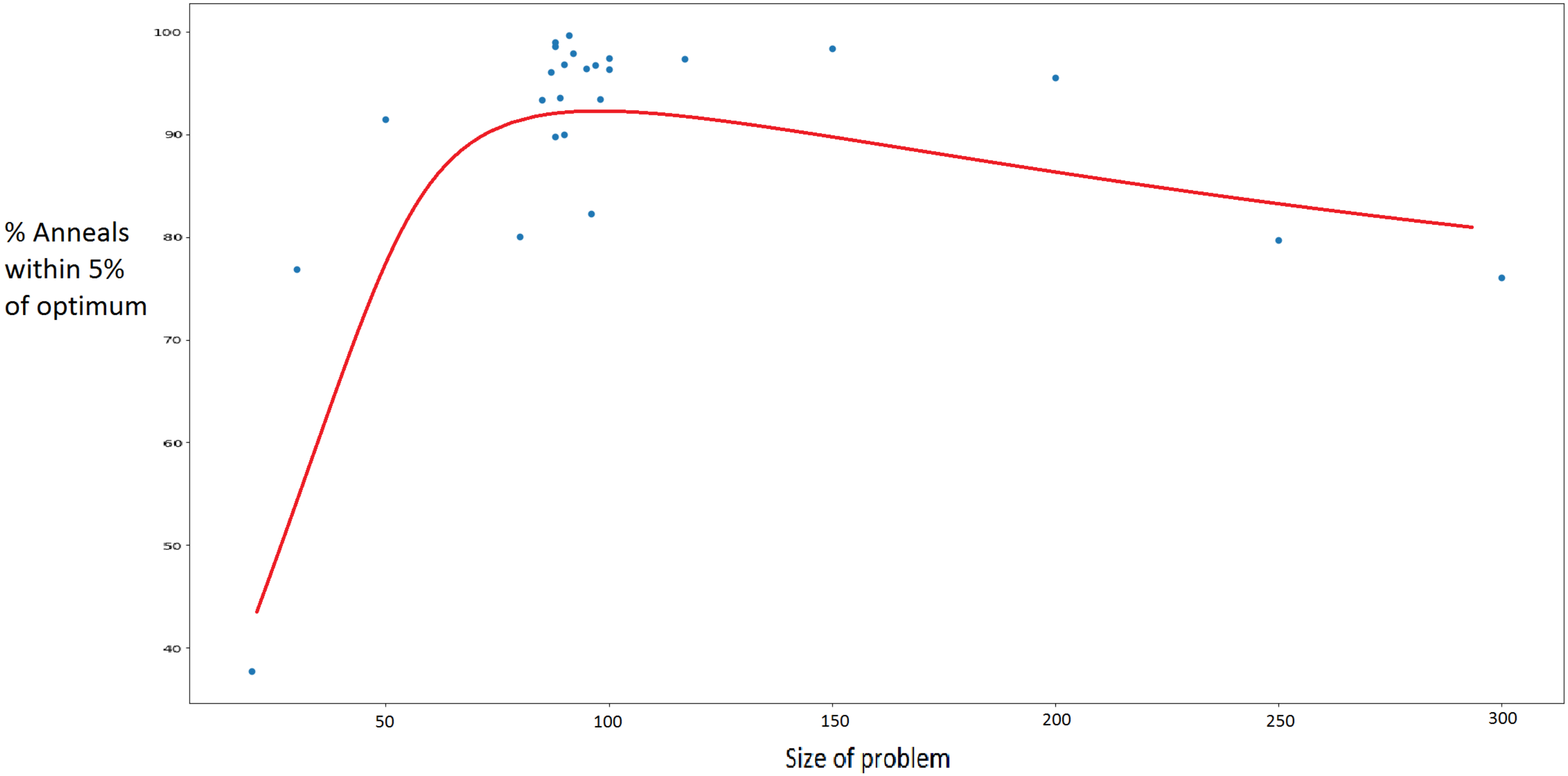
Optimum solution for a 300 node graph, found by D-Wave and verified classically.

Finding Exact Optimum



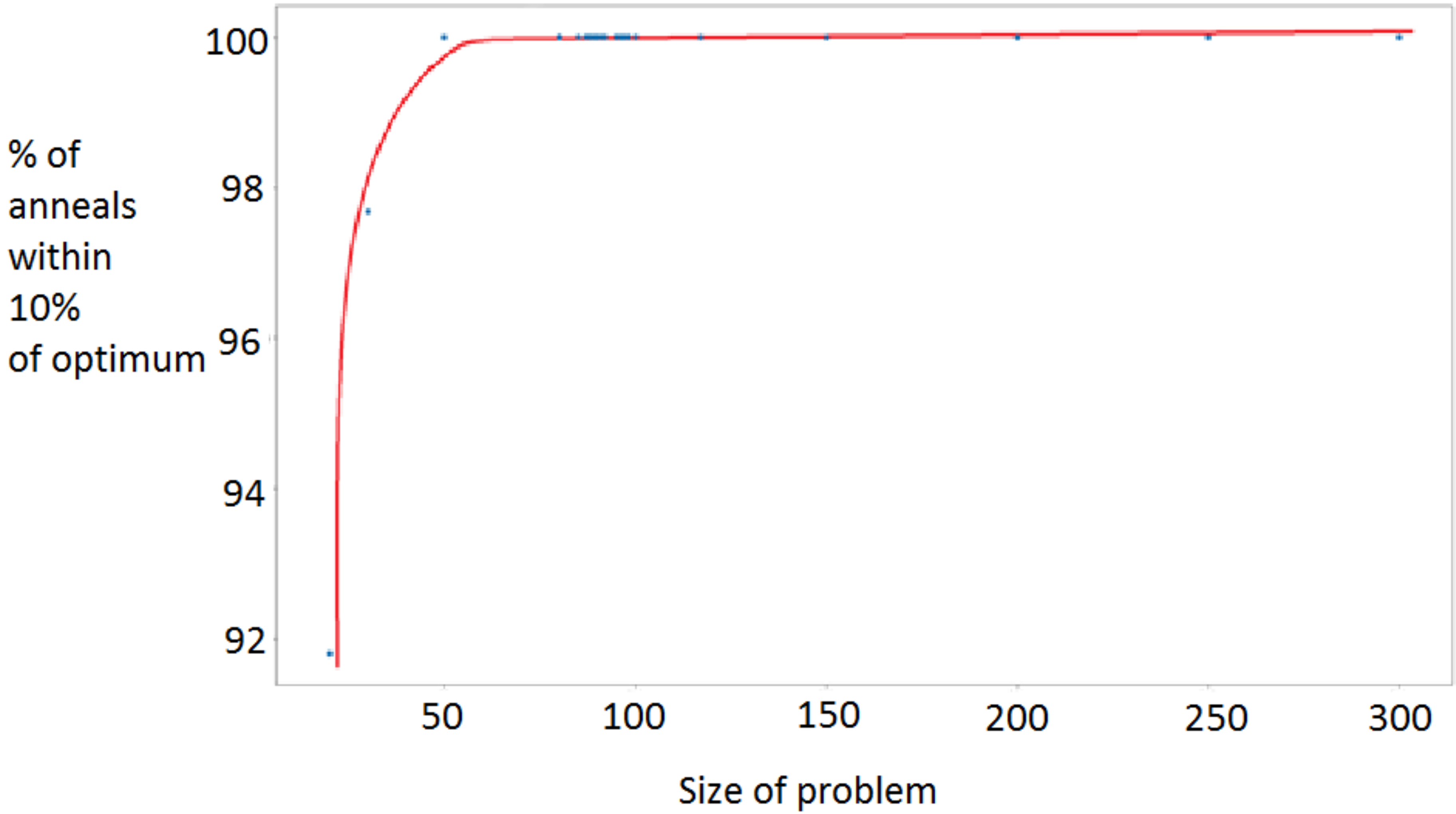
- Percentage of anneals that return exact optimum over a range of problem sizes between 20 and 100 logical cells.

Finding Near Optimum (>95% of optimum)



- Percentage of anneals that return >95% optimum over a range of problem sizes between 20 and 300 nodes.

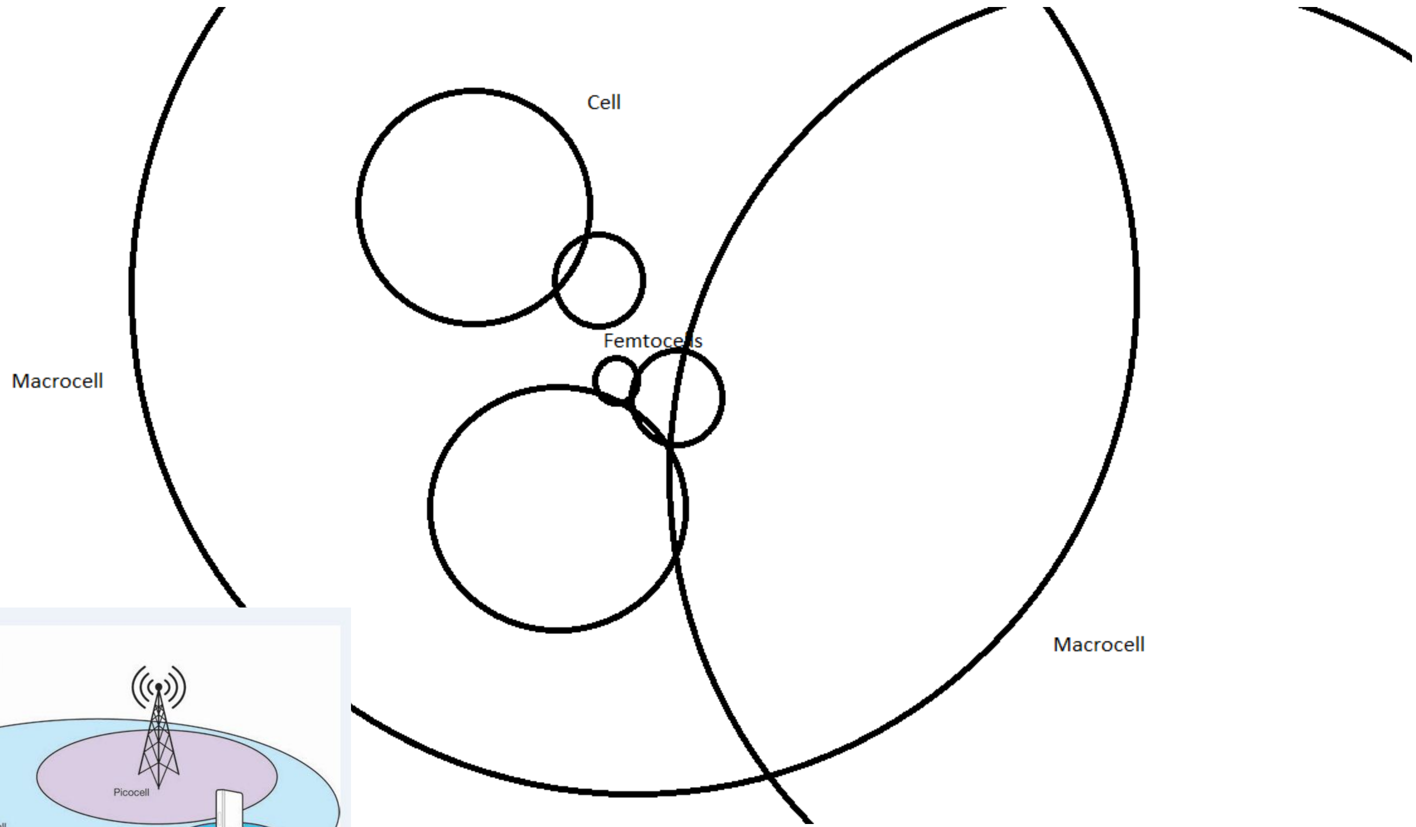
Finding Near Optimum (>90% of optimum)

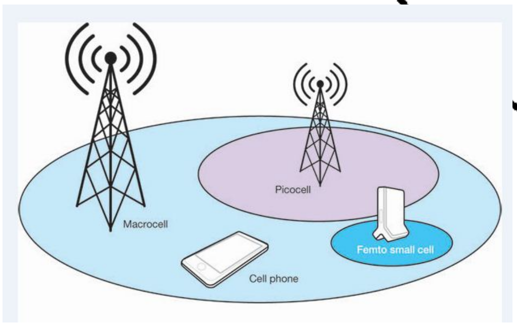


- Percentage of anneals that return >90% optimum over a range of problem sizes between 20 and 300 nodes.

Best allocation n-colouring

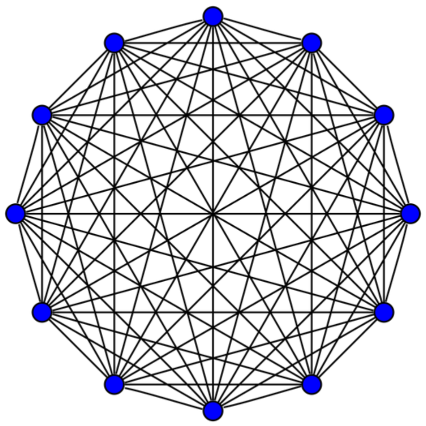
- When colouring a graph with a large number of colours, (frequencies) and varying demand – how best to allocate channels to satisfy demand?





Cell Channel Allocation Problem

1. Each cell is represented by a complete graph of qubits
- one qubit for each available channel.



- Demand on each cell can be mapped onto the complete graph. That is, we set the coupling values such that the minima of the isolated Hamiltonian corresponds to a channel allocation that meets but does not exceed the ideal demand, for n channels.

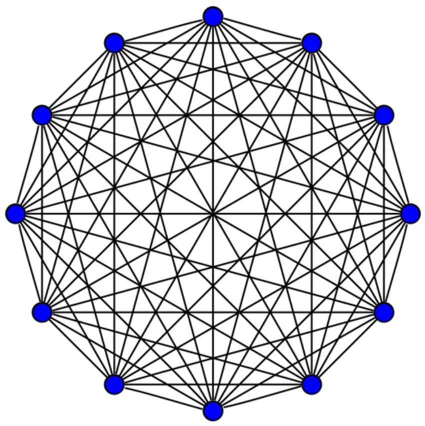
- To express the objective that optimum available channels to meet demand is N
- $Minimise\ ((\sum Q_i) - N)^2$

Cell Channel Allocation Problem

Qubo terms, where N_i is the channel demand of cell *i*:

Transform QUBO $y_i \in \{0,1\}$ to Ising $s_i \in \{-1,1\}$ form:

- $Minimise \left((\sum Q_i) - N_i\right)^2$



- Transformation: $y_i = \frac{s_i+1}{2}$

Substitute this into the QUBO

- $Minimise \left(\left(\sum \frac{s_i+1}{2}\right) - N_i\right)^2$

Expand, and note that for both $s_i = 1$ and $s_i = -1$, $s_i^2 = 1$

- $Minimise \left(\sum_{i \neq j} s_i * s_j + \sum_i (M - 2N) s_i + const\right)$ where *M* is number of colours

Constant terms can be dropped

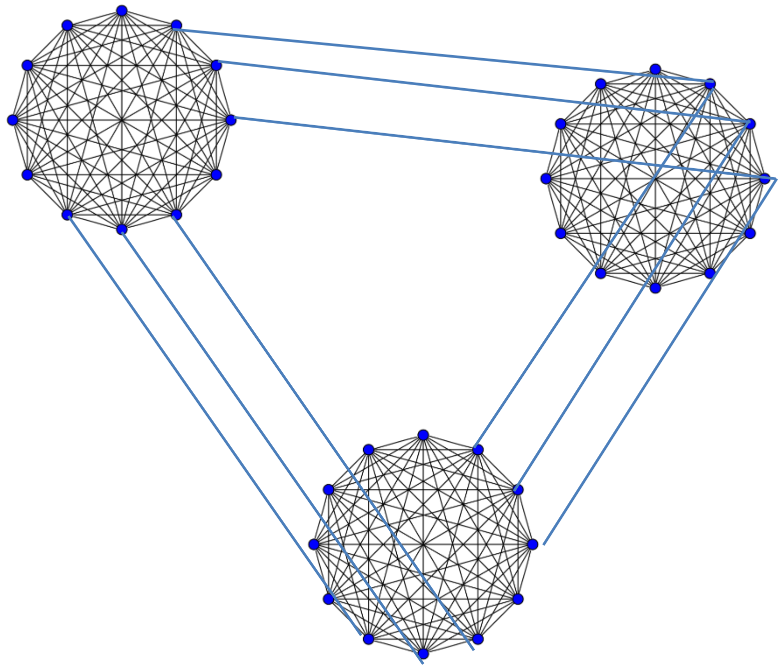
$$Minimise \left(\sum_{i \neq j} s_i * s_j + \sum_i (M - 2N_i) s_i\right)$$

- We can add further details such as penalising onsite energies corresponding to frequencies that don't perform well for that cell.

Cell Channel Allocation Problem

2.

The constraint on neighbouring cells not taking the same frequency
is obtained by a 1:1 mapping between the same channels between graphs,
which we couple with a strong antiferromagnetic value (J>1).



Small Tests of Cell Allocation Problem: 3 cells, even distribution

Each cell:

$M = 3,\ N = 1\ ,\ M - 2N = 1,$

$i, j \in \{0,1,2\}$

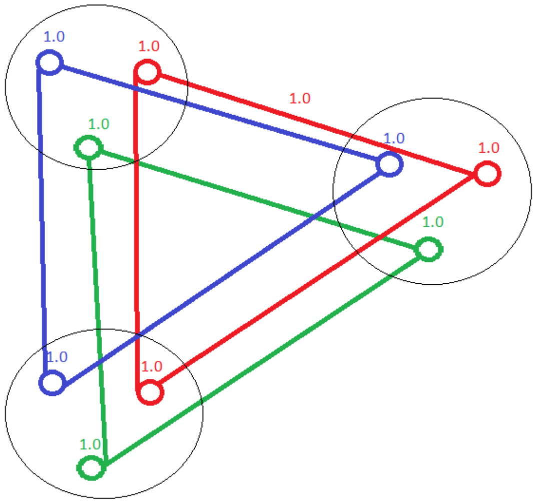
$$\rightarrow \left(\sum_{i \neq j} s_i * s_j + \sum_i s_i \right)$$

Cell interference graph terms:

$K_{IJ} = 1\ if\ cells\ I\ and\ J\ interfere$

$I, J \in \{Cells\}$

$$\rightarrow \left(K_{IJ} \sum_{I \neq J} \sum_{c=0..M} s_{I+c} * s_{J+c} \right)$$



Overall:

$$\text{Minimise} \left(\sum_{i \neq j} s_i * s_j + \sum_i s_i + K_{IJ} \sum_{I \neq J} \sum_{c=0..M} s_{I+c} * s_{J+c} \right)$$

Small Tests of Cell Allocation Problem: 4 cells, one with high demand

$$M = 5,\ N_{I=0} = 2,\ N_{I\in\{1,2,3\}} = 1\ ,\ M - 2N_{1,2,3} = 3,\ M - 2N_0 = 1$$

$$i, j \in \{0,1,2,3,4\}$$

Cell with high demand $\left(\sum_{i\neq j} s_i * s_j + \sum_{i\in\{1,2,3\}} s_i\right)$

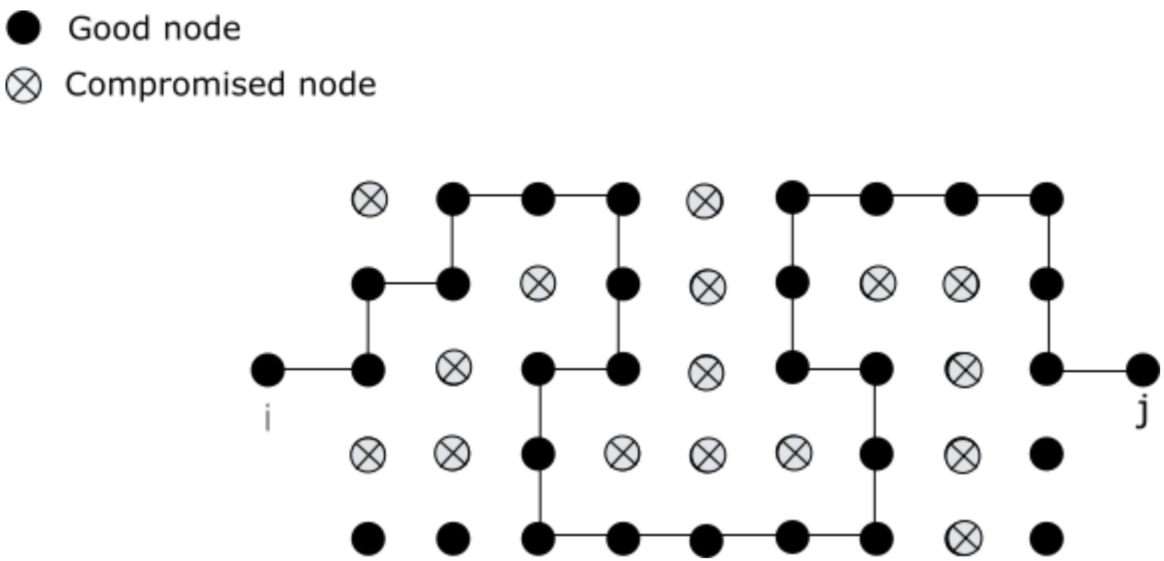
Cells with normal demand $\left(\sum_{i\neq j} s_i * s_j + 3\sum_{i\in\{1,2,3\}} s_i\right)$

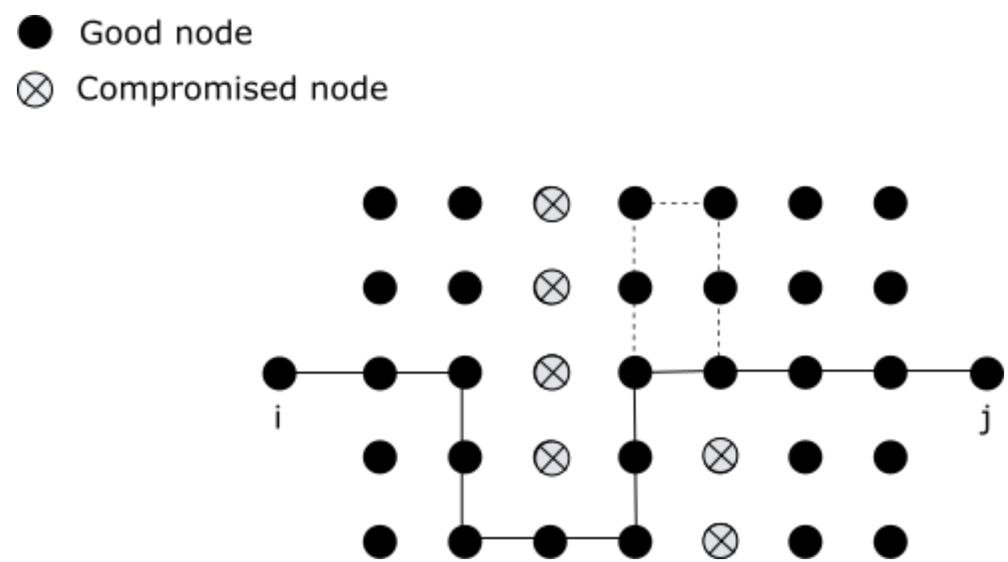
Adjust the previous problem to penalise use of channels which are 'bad' for the cell, by use of a quality factor q_i.

Cell with high demand $\left(\sum_{i\neq j} s_i * s_j + q_i \sum_{i\in\{1,2,3\}} s_i\right)$

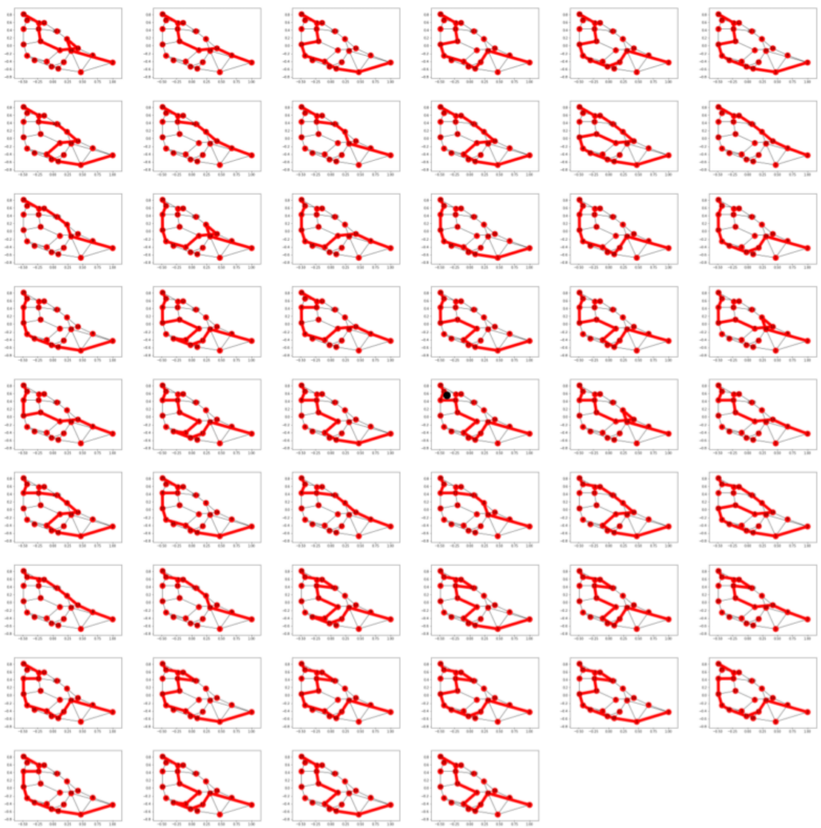
Cells with normal demand $\left(\sum_{i\neq j} s_i * s_j + q_i 3\sum_{i\in\{1,2,3\}} s_i\right)$

Vertex-diverse routing





Partially vertex disjoint paths on a ‘core’ model



Vertex-disjoint routing: Useful problems to solve

- What is the best set of partially disjoint paths between nodes *s* and *t*?
- What if... the probability of node compromise is different for each node?
- What if... we group the nodes into 'shared risk groups' which will all be compromised together?
- What if…we want to load balance across all a subset of nodes (which we define as the network edges) – and we want to find an optimum set of partially diverse routes between all pairs in this set, and we make the rule that nodes cannot be shared?

...and we can look at all the same problems, but for edge-disjointness

Hard problems we are trialling with DWave

- Half duplex mesh network
- Cell channel allocation
- Routing and Wavelength Assignment
- Network resilience – disjoint path routing
- Job shop scheduling
- Malicious traffic flow propagation and defensive strategies

Conclusions

- D-Wave **reliably generates near optimums using a small number of anneal cycles**.
- Many discrete optimisation problems from the telecommunication industry **map very well** to the D-Wave
- If this performance can be maintained for larger processors, D-Wave will be a **significant technology** for this industry.
- **Chain-length minimisation** is a big issue. Hierarchical **connectivity** or bespoke architectures could be an interesting approach.
- Suggestion: D-Wave could make their built-in functions very flexible, i.e. provide variations on Graph Colouring to allow n-color allocation, and to provide preference on allocated color.

Plantagenet SYSTEMS BT UCL University of BRISTOL D-Wave The Quantum Computing Company™

Thanks

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